Temperature from quantum entanglement

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[1] with S. Shankaranarayanan(based on arXiv: 1504.00501)

Outline of the talk

- Introduction
- Black hole mechanics
- Entanglement -An approach to find out entropy
- The famous “Area law”
- Motivation towards the problem
- How to compute EE ?
- Comparison of entanglement temp. with BH temp.
- Conclusions

Black hole mechanics

- **A black hole is a region of space-time from which gravity prevents anything, including light, from escaping.**

J.D.Bekenstein, Phys.Rev.D **7**,2333 (1973)
S.W.Hawking, Commun. math. Phys. 31, 161-170 (1973)

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- Classically it has infinite entropy and zero temperature and **obey all laws of black hole mechanics**

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$$d(Mc^2) = \frac{\kappa c^2}{8\pi G} dA + \Omega dJ + \Phi dQ$$

$$\frac{dA}{dt} \geq 0$$

Semi classically it has finite entropy and temperature.

$$T_{BH} = \frac{\hbar c^3}{G k_B} \frac{1}{8\pi M} \; ; \quad S_{BH} = \frac{k_B c^3}{4} \frac{A_H}{G\hbar}$$

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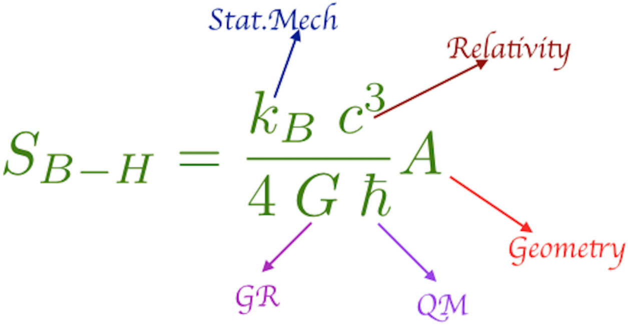
Issues related to black-hole mechanics



$$S_{B-H} = \frac{k_B\, c^3}{4\, G\, \hbar} A$$

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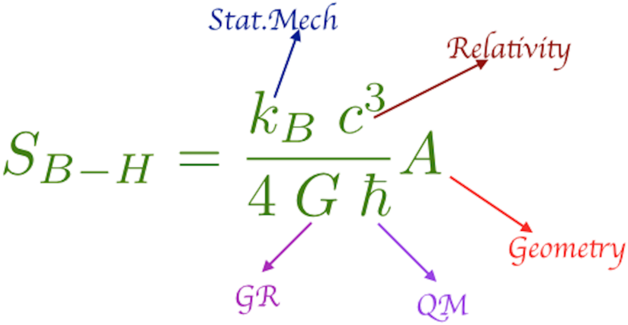

- **What is the statistical mechanical interpretation for the black-hole entropy?**
- **Can we have an approach which can give Bekenstein-Hawking entropy and Hawking temperature?**

Quantum entanglement- Bipartite

- Two spin-1/2 particles

$$|\Psi\rangle = \cos\theta \,|\uparrow\rangle_A |\downarrow\rangle_B + \sin\theta \,|\downarrow\rangle_A |\uparrow\rangle_B$$

  are entangled

- A quantum system is in a pure state $|\Psi\rangle$, the density matrix is

$$\rho = |\Psi\rangle\langle\Psi| \quad (\mathrm{tr}\rho = 1)$$

- $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

Entanglement measure - von Neumann entropy

The reduced density matrix for each part is

$$\rho_A = \mathrm{Tr}_B\, \rho\,; \quad \rho_B = \mathrm{Tr}_A\, \rho$$

The von-Neumann Entropy is

$$S_A = -\mathrm{Tr}(\rho_A\ \ln \rho_A) = -\sum_k \lambda_k \ln \lambda_k = S_B$$

λ_k are the eigenvalues of ρ_A or ρ_B

- **Entanglement Entropy= von-Neumann Entropy —Non-extensive**

$$S_\alpha^{\textbf{Rényi}} = \frac{\textbf{log Tr}\rho^\alpha}{1-\alpha}, \qquad S_A = \lim_{\alpha \to 1} S_\alpha$$

The famous “Area law”

The leading divergent term of EE in a (D+1) dim. QFT is proportional to the area of the (D-1) dim. boundary

$$S_A = \frac{\textbf{Area}}{a^{D-1}} + \textbf{sub-leading terms}$$

where a is the UV cut off (lattice spacing)

Srednicki, Phys.Rev.Lett. **71**,666 (1993)

Bombelli et al, Phys.Rev.D **34**,373 (1986)

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The area law resembles the Bekenstein-Hawking formula of black hole entropy:

$$S_{BH} = \frac{\text{Area of horizon}}{4G}$$

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Bombelli et al, Phys.Rev.D **34**,373 (1986)

Comments on Area law

- **The area law is satisfied for ground state (both massless and masisve theory). It violates for excited states, Power law!** [Das, Shankaranarayanan, and Sur, 08]
- **It is showed that the EE in higher dimensions is proportional to the higher dimensional area using Rényi entropy as a measure.** [Braunstein, Das, and Shankaranarayanan,13]

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The motivation

Is there any way of obtaining microcanonical temperature from entanglement entropy which identical to the Hawking temperature and it satisfies the first law of black-hole thermodynamics?

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Is there any way of obtaining microcanonical temperature from entanglement entropy which identical to the Hawking temperature and it satisfies the first law of black-hole thermodynamics?

The answer is YES!

Our approach to the Motivation- To compute EE?

- Uses the QFT techniques in real time (Direct method) to calculate EE
- The spherically symmetric Lemaître black hole space- time (τ, ξ) is used for the calculation

$$ds^2 = d\tau^2 - (1 - f[r(\tau, \xi)])\, d\xi^2 - r^2(\tau, \xi) d\Omega_D^2$$

- Advantage of Lemaître over Schwarzschild space-time :
  - Removes the coordinate singularity at the horizon
  - The coordinate τ is time like every where, while ξ is space like.
  - Relation to Schwarzschild radius

$$\xi - \tau = \int \frac{dr}{\sqrt{1 - f[r]}}$$

  - Time dependence in τ allows to do the computation at different Lemaître times.
  - The specific choices of $f(r)$ leads to different BH space-time

- **The action for the massless scalar field $\Phi(x^\mu)$ in $D+2$ dimensional space-time is**

$$\mathcal{S} = \frac{1}{2}\int \sqrt{-g}\, g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\, d^{D+2}\mathbf{x}$$

- **The metric has spherical symmetry,**

$$\Phi(\mathbf{x}) = \sum_{l,m_i}\Phi_{lm_i}(\tau,\xi)Z_{lm_i}(\theta,\phi_i)$$

where i $\in \{1,2,\ldots D-1\}$.

- **do the following infinitesimal transformations,**

$$\tilde{\tau} \to \tilde{\tau}' = \tilde{\tau} + \epsilon, \qquad \tilde{\xi} \to \tilde{\xi}' = \tilde{\xi}$$
$$\tilde{\Phi}_{lm_i}(\tilde{\tau},\tilde{\xi}) \to \tilde{\Phi}'_{lm_i}(\tilde{\tau}',\tilde{\xi}') = \tilde{\Phi}_{lm_i}(\tilde{\tau},\tilde{\xi})$$
$$\tilde{r}(\tilde{\tau}',\tilde{\xi}') = \tilde{r}(\tilde{\tau}+\epsilon,\tilde{\xi})$$

Perturbed scalar field Hamiltonian

- The perturbed Hamiltonian is

$$
\begin{aligned}
H \quad & \simeq \frac{1}{2}\sum_{l,m_i}\int_{\tilde{\tau}}^{\infty} d\tilde{r}\left[\pi^2_{lm_i} + \tilde{r}^D \frac{\left(1-\epsilon H_1 - \epsilon^2 H_2\right)^D}{\left(1+\epsilon H_3 - \epsilon^2 H_4\right)^{1/2}}\right. \\
& \times \left[\partial_{\tilde{r}} \frac{\sigma_{lm_i}}{\tilde{r}^{D/2}\left(1-\epsilon H_1 - \epsilon^2 H_2\right)^{D/2}\left(1+\epsilon H_3 - \epsilon^2 H_4\right)^{1/4}}\right]^2 \\
& \left. + \frac{l(l+D-1)}{\tilde{r}^2\left(1-\epsilon H_1 - \epsilon^2 H_2\right)^2}\sigma^2_{lm_i}\right]
\end{aligned}
$$

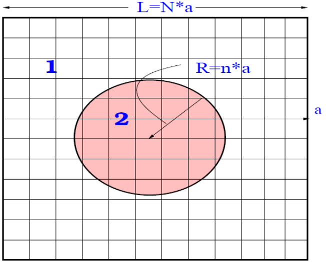
  This is a free field Hamiltonian propagating in flat spce time at fixed Lemaître time $\tilde{\tau}$.

- Fields obey the usual commutation relations

$$
\left[\pi_{lm_i}(\tilde{r},\tilde{\tau}), \sigma_{l'm'_i}(\tilde{r}',\tilde{\tau})\right] = i\delta_{ll'}\delta_{m_i m'_i}\delta(\tilde{r}-\tilde{r}')
$$

- Calculate the EE at each ϵ slices by setting $\tilde{\tau} = 0$

- The measure used is von-Neumann entropy for lower space-time dimensions and Rényi entropy for higher dimensions
- Using central difference scheme discretization, $H \Longrightarrow$ **System of coupled HO**
- Integrate out the DOF in spatial region **1**. remaining DOF are described by a density matrix $\rho_2 \Longrightarrow$ EE. The accuracy is 10^{-8} and $N = 300$



Srednicki, Phys.Rev.Lett. **71**,666 (1993)
Braunstein, Das, and Shankaranarayanan, JHEP 1307 (2013) 130

Important observations

- **At $\epsilon = 0$, leads to the flat space- time dimensional Hamiltonian**
- **At all times, the EE satisfies the area law**
- **Perturbation over ϵ allows to calculate EE (set $10 \leq n \leq 150$ and take average) and total internal energy (E) as a function of ϵ**
- **The system considering is micro canonical, define the entanglement temperature as**

$$T_{EE} = \frac{\Delta E}{\Delta S} = \frac{\textbf{Slope of the energy w.r.t } \epsilon(\Delta E/\Delta\epsilon)}{\textbf{Slope of the EE w.r.t } \epsilon(\Delta S/\Delta\epsilon)}$$

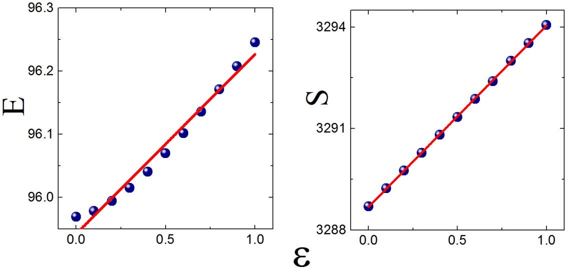
- **Black hole temperature**

$$T_{BH} = \frac{\kappa}{2\pi} = \frac{1}{2}\frac{df(\tilde{r})}{d\tilde{r}}\Big|_{\tilde{r}=R_h}$$

4D Schwarschild BH's

$$f(\tilde{r}) = 1 - \frac{1}{\tilde{r}}$$

$\tilde{r} = r/R_h$ is the rescaled radius w.r.t the horizon radius $R_h(= 2M)$.



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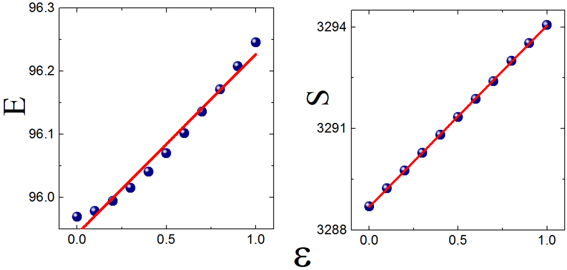
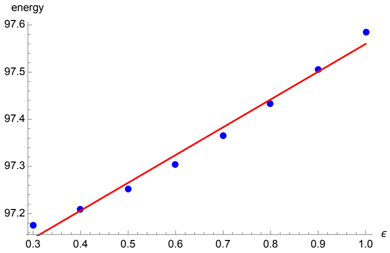
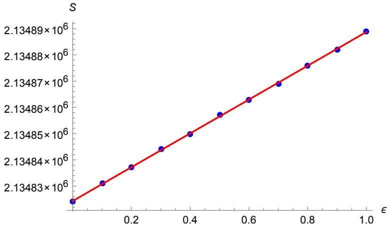


Both black hole entropy and internal energy increasing as a function of ϵ

The exact result is $T_{BH} \sim 0.079$ and we got numerically $T_{EE} \sim 0.076$

6D Schwarschild BH entropy



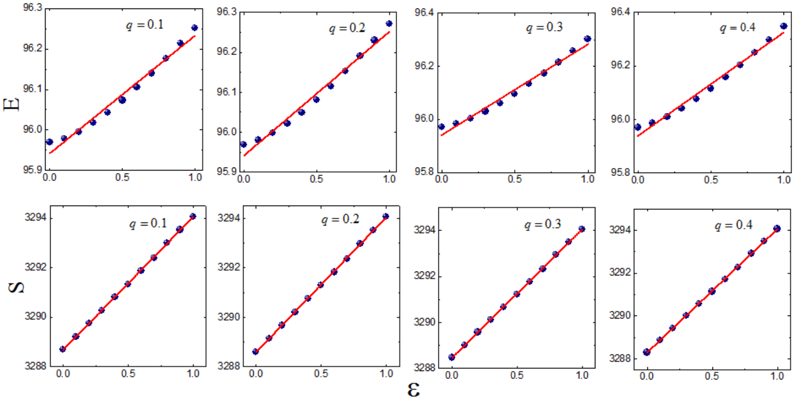


The exact result is $T_{BH} \sim 0.238$ and we got numerically $T_{EE} \sim 0.203$

4D Reissner-Nordstrm BH's

$$f(\tilde{r}) = 1 - \frac{2M/R_h}{\tilde{r}} + \frac{(Q/R_h)^2}{\tilde{r}^2} \quad (2)$$

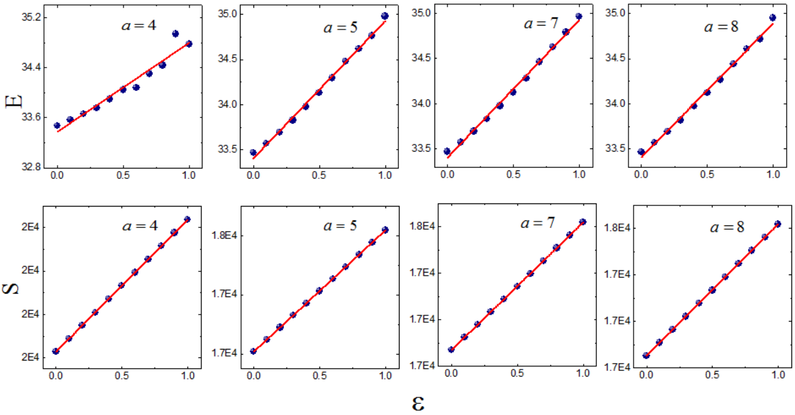
where Q is the charge of the black hole. Rescale the radius w.r.t the outer horizon ($R_h = M + \sqrt{M^2 - Q^2}$).



The exact result is $T_{BH} \sim 0.0787$ and we got numerically $T_{EE} \sim 0.0776$ for $q = 0.1$

6D Boulware- Deser BH's

$$f(\tilde{r}) = 1 + \frac{\tilde{r}^2 \tilde{R}_h^2}{6a}\left(1 - \sqrt{1 + \frac{12a}{\tilde{r}^5 \tilde{R}_h^5}}\right)$$



The exact result is $T_{BH} \sim 0.039$ and we got numerically $T_{EE} \sim 0.044$ for $a = 4$

D. G. Boulware and S. Deser, 1985

Comparision of T_{EE} & T_{BH}

Black hole		$(\Delta S/\Delta\epsilon)$	$(\Delta E/\Delta\epsilon)$	T_{BH}	T_{EE}
Schwarzschild		3.729	0.2846	0.07958	0.07632
Schwarzschild-6D		31.01	0.6297	0.2387	0.2031
R-N	$q = 0.1$	3.747	0.2909	0.07878	0.07764
	$q = 0.2$	3.801	0.3096	0.07639	0.08145
	$q = 0.3$	3.891	0.3414	0.07242	0.08774
	$q = 0.4$	4.011	0.3868	0.06685	0.09643
B-D	$a = 4$	320	1.422	0.03985	0.0444
	$a = 5$	373	1.518	0.03982	0.0408
	$a = 7$	479	1.52	0.0398	0.0317
	$a = 8$	552	1.479	0.0398	0.0268

Numerically both temperature matches!

S.K.S and S. Shankaranarayanan, arXiv: 1504.00501

Conclusions

- **Numerically, the temperature predicted by the black hole mechanics is matches approximately with the entanglement temperature**
- **This is the important numerical evidence to the first laws of BH mechanics (at constant charge, Gauss-Bonnet coupling term) from quantum information**
- **This may underline a strong connection between the Bekenstein-Hawking entropy and the entanglement entropy**

Thank you for your kind attention.